Branes and Fluxes in $D = 5$ Calabi-Yau Compactifications of M-Theory

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We discuss Poincaré three-brane solutions in $D = 5$ M-Theory compactifications on Calabi-Yau (CY) threefolds with G-fluxes. We show that the vector moduli freeze at an attractor point. In the case with background flux only, the spacetime geometry contains a zero volume singularity with the three-brane and the CY space shrinking simultaneously to a point. This problem can be avoided by including explicit three-brane sources. We consider two cases in detail: a single brane and, when the transverse dimension is compactified on a circle, a pair of branes with opposite tensions.

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1. Introduction and Summary

One of the most challenging problems in string theory is to uncover the vacuum selection mechanism. Assuming that this mechanism can be understood within the framework of low-energy field theory, the problem amounts to the computation of the effective potential for the moduli fields. For quite a long time, non-perturbative effects, like gaugino condensation, have been considered (with limited success) as a possible source of such potentials. More recently, starting with the work of Polchinski and Strominger [1], the research focus has shifted to compactifications involving non-vanishing background fluxes of various antisymmetric tensor fields – so-called G-fluxes [2-5]. In fact, for Calabi-Yau compactifications of type II theory, G-fluxes can generate [6] the most general form of the (super)potential allowed by $N = 2$ supersymmetry. In five dimensions, in Calabi-Yau compactifications of M-Theory, G-fluxes of the eleven-dimensional three-form gauge field produce a similar potential [7,8,9]. This type of compactification is particularly interesting, since M-Theory provides a powerful setup for studying string dynamics.

It is interesting to look at so-called warped compactifications with Poincaré invariance in this context. Since string theory is known to contain higher dimensional extended objects in an essential way, it is natural to look at compactifications which involve them in a nontrivial fashion. Recently, some examples of this type have been studied in connection with the hierarchy problem [10] and with the cosmological constant problem [11].

In this work, we study the classical field equations of M-Theory compactified from $D = 11$ to $D = 5$ on Calabi-Yau (CY) threefolds with various G-flux configurations. In the absence of fluxes, the effective field theory is $D = 5$ supergravity [12] coupled to a number of vector and hyper multiplets (as determined by the cohomology of the Calabi-Yau space [13]). The presence of fluxes results in gauged supergravity [12] with a non-vanishing potential [7,8,9].

We first discuss the case of smooth background fluxes, i.e. without explicit sources. We consider a Poincaré three-brane solution of the form

$$ds^2 = e^{2\phi(u)}\eta_{\alpha\beta}dx^\alpha dx^\beta + du^2 \,, \tag{1.1}$$

where the x-coordinates parameterize the $D = 4$ three-brane world-volume, $\eta_{\alpha\beta}$ is the flat four-dimensional Minkowski metric, while u is the "fifth" coordinate (transverse to the three-brane). The Weyl factor $e^{2\phi(u)}$ depends only on the transverse coordinate u and is related by the field equations to the CY volume. The solution (1.1) exhibits a zero

volume singularity with the three-brane and the CY threefold shrinking simultaneously to a point. On the other hand, the shape of the Calabi-Yau manifold, determined by its vector (Kähler) moduli, remains frozen at a point corresponding to the extremal value of the central charge. In fact, the stability condition turns out to be exactly the same as the attractor equation [14,15] for a $D = 5$ black hole, with the charges identified as G-fluxes.

The zero volume singularity can be avoided by introducing a G-flux discontinuity across a three-brane source. While the detailed interpretation of the sources is beyond the scope of this paper, the required tension (determined by the equations of motion) indicates the presence of a fivebrane wrapped on a Calabi-Yau two-cycle. In the case of a compact transverse dimension, we construct flux configurations supported entirely by a pair of effective brane sources with opposite tensions. This system is somewhat similar to the one considered by Randall and Sundrum [10]. In the present case, however, the bulk spacetime is not AdS.

The paper is organized as follows. In Section 2, we establish notation and review $D = 5$ M-Theory CY compactifications with G-fluxes. The Poincaré three-brane solution is presented in Section 3. In Section 4, we establish the connection with the attractor mechanism. We introduce flux sources in Section 5. In Section 6, we examine the supersymmetry variations of fermions and identify the unbroken supersymmetry transformations. Section 7 contains conclusions and outlook.

2. CY Compactification of M-Theory with Background Fluxes

This section is a brief review aimed at fixing notation. The compactification of $D = 11$ supergravity on a Calabi-Yau threefold with Hodge numbers $(h_{1,1}, h_{2,1})$ results in an $N = 2$, $D = 5$ supergravity theory interacting with $h_{1,1} - 1$ vector multiplets and $h_{2,1} + 1$ hypermultiplets [13]. In our discussion, hypermultiplets play no role, except for the universal hypermultiplet involving the CY volume. The relevant part of the action is determined by a cubic prepotential $\mathcal{V}$ which is fixed by the CY intersection numbers. Details of this can be found in a number of references, see e.g. [13,16]. Modifications arising from the presence of background fluxes of the four-form field-strength have been discussed in [7,8] and more recently recently in [9]. The presence of a background flux implies that the supergravity is gauged [12], and a potential of a specific form is induced in the five-dimensional effective action.

Let us denote by M^i the scalar Kähler moduli of the Calabi-Yau threefold so that the Kähler form

$$J = M^i \omega_i \,, \tag{2.1}$$

where ω_i, $i = 1, \ldots, h_{1,1}$ is a basis of $H^{(1,1)}$ two-forms, and the CY volume

$$\mathcal{V}(M) = \frac{1}{6} \int_{CY} J \wedge J \wedge J = \frac{1}{6} c_{ijk} M^i M^j M^k \,, \tag{2.2}$$

where c_{ijk} are the intersection numbers. In the absence of background fluxes the action[1] is given by [13]

$$S_0 = \int d^5x \sqrt{-g} \left[\frac{1}{2} \mathcal{V} R + \frac{1}{2} (\mathcal{V} G_{ij} + \partial_i \partial_j \mathcal{V}) \, \partial_\mu M^i \partial^\mu M^j + \ldots \right] \tag{2.3}$$

where the moduli space metric is

$$G_{ij}(M) = \frac{i}{2\mathcal{V}} \int_{CY} \omega_i \wedge {}^\star \omega_j = -\frac{1}{2} \partial_i \partial_j \ln \mathcal{V} \,. \tag{2.4}$$

In the above equations, $\partial_i \equiv \frac{\partial}{\partial M^i}$. It is often convenient to parameterize the moduli space in a way that makes manifest the decoupling of vector multiplets and hypermultiplets. This entails a Weyl rescaling of the metric as well as introducing the special coordinates $X^i = M^i \mathcal{V}^{-1/3}$, and treating the volume (2.2) as an independent field belonging to the universal hypermultiplet. For the present purpose this is not so useful, so the volume $\mathcal{V}$ will be regarded as a function of the moduli as given in (2.2).

The presence of background fluxes gives rise to a potential [7,8,9]. We will consider the following four-form field strength of the three-form gauge field:

$$G_{\text{flux}} = \frac{i}{2\mathcal{V}} \alpha_i G^{ij} \, {}^\star \omega_j \,, \tag{2.5}$$

where α_i are integers, as required by flux quantization conditions, and G^{ij} is the inverse of the moduli space metric (2.4). The scalar potential originates from $D = 11$ kinetic terms, which upon compactification yield the term

$$S_{\text{flux}} = -\frac{1}{8} \int d^5x \sqrt{-g} \, \mathcal{V}^{-1} G^{ij} \alpha_i \alpha_j \,. \tag{2.6}$$

Equations (2.3) and (2.6) are written in the string frame. In order to obtain the canonical Einstein-Hilbert term one performs the Weyl rescaling

$$ds_{\text{E}}^2 = \mathcal{V}^{2/3} ds^2 \,. \tag{2.7}$$

In the Einstein frame, the full action, $S = S_0 + S_{\text{flux}}$, reads

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} G_{ij} \partial_\mu M^i \partial^\mu M^j - \frac{1}{6} \partial_\mu (\ln \mathcal{V}) \partial^\mu (\ln \mathcal{V}) - \frac{1}{8} \mathcal{V}^{-8/3} G^{ij} \alpha_i \alpha_j + \ldots \right] \,. \tag{2.8}$$

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[1] In our conventions, the metric has signature $(-++++)$ and the Ricci tensor $R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\mu\nu} + \ldots$

3. The Solution

In this section, we solve the classical field equations for the extremum of the action (2.8). We look for a gravitational background of the form (1.1) representing a Poincaré-symmetric three-brane in five dimensions. The non-vanishing components of the corresponding Einstein tensor are

$$E_{\alpha\beta} = \eta_{\alpha\beta} e^{2\phi} [3\phi'' + 6(\phi')^2] \, ,$$
$$E_{uu} = 6(\phi')^2 \, , \tag{3.1}$$

where the prime denotes a derivative with respect to the transverse coordinate u.

The initial observation is that the variation of the action with respect to the moduli M contains the terms

$$\delta S = \int d^5 x \sqrt{-g} [-\frac{1}{2} \partial_\mu M^i \partial^\mu M^j \delta G_{ij} + \frac{1}{8} \mathcal{V}^{-8/3} \alpha_i \alpha_j G^{im} G^{jn} \delta G_{mn} + \ldots] \tag{3.2}$$

which suggests considering solutions with moduli depending only on u (in line with the Poincaré symmetry of the metric (1.1)), together with a BPS-like Ansatz

$$2(M^i)' = \mathcal{V}^{-4/3} G^{ij} \alpha_j \, . \tag{3.3}$$

This Ansatz leads to several simplifications. First of all, the (uu) component of Einstein's equations becomes

$$6(\phi')^2 = \frac{1}{6} (\frac{d \ln \mathcal{V}}{du})^2 \, . \tag{3.4}$$

This is solved by

$$e^{2\phi} = e^{2\phi_0} \, \mathcal{V}^{1/3} \, , \tag{3.5}$$

where ϕ_0 is a constant. We will ignore the second solution, $e^{2\phi} \propto \mathcal{V}^{-1/3}$, since it fails to satisfy some other field equations; we will comment on this below. The $(\alpha\beta)$ components of Einstein's equations simplify after using Eqs. (3.3) and (3.5). They become:

$$(\mathcal{V}^{2/3})'' + \frac{2}{3} \mathcal{V}^{-2/3} (\alpha_i M^i)' = 0 \, . \tag{3.6}$$

The remaining terms in the variation of the action with respect to the moduli, after substituting Eqs.(3.3) and (3.5), lead to the following equations:

$$(\mathcal{V}^{-2/3})' \alpha_i + \mathcal{V}^{-1} (\mathcal{V}^{2/3})'' \partial_i \mathcal{V} + \frac{4}{3} \mathcal{V}^{-5/3} (\alpha_k M^k)' \partial_i \mathcal{V} = 0 \, . \tag{3.7}$$

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For this to have a solution it must be the case that α_i is parallel to $\partial_i \mathcal{V}$. Thus it is natural to look for a solution such that

$$\alpha_i = \frac{\zeta}{3} \mathcal{V}^p \partial_i \mathcal{V} , \tag{3.8}$$

where ζ and p are constants. It would seem that the two Ansatze (3.3) and (3.8) impose too many constraints; fortunately, this is not the case. First, by checking the compatibility of Eq.(3.8) with Eqs.(3.3) and (3.6) we find[2] that the power $p = -2/3$. In this way, Eq.(3.8) becomes

$$\mathcal{V}^{-2/3} \partial_i \mathcal{V} = \frac{3\alpha_i}{\zeta} . \tag{3.9}$$

In the process, we also solve the second Einstein equation (3.6), with the result:

$$\mathcal{V} = \mathcal{V}_0 + \zeta u , \tag{3.10}$$

where $\mathcal{V}_0$ is a constant. Finally, we use Eqs.(3.9) and (3.10) to verify that the moduli equation of motion (3.7) is indeed satisfied. In this way, Eqs.(3.9) and (3.10) together with Eq.(3.5) yield a consistent solution of all field equations.

A few remarks are in order here. Note that by using the formulae of the previous section, Eq.(3.9) can be rewritten in a more geometric way as an equation[3] describing the flux (2.5):

$$G_{\text{flux}} = \frac{\zeta}{6} \mathcal{V}^{-2/3} J \wedge J . \tag{3.11}$$

Furthermore, since Eq.(3.9) is invariant under the rescaling $M^i \to \lambda M^i$ with an arbitrary constant λ, it is possible to rewrite it exclusively in terms of the special coordinates

$$X^i(M) = M^i \mathcal{V}^{-1/3} , \tag{3.12}$$

which satisfy the constraint

$$\mathcal{V}(X) = \frac{1}{6} c_{ijk} X^i X^j X^k = 1 . \tag{3.13}$$

Recall that special coordinates parameterize the $(h_{1,1}-1)$-dimensional vector multiplet space [12]. In this way, one obtains

$$c_{ijk} X^j X^k = \frac{6\alpha_i}{\zeta} . \tag{3.14}$$

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[2] This is the point where the second solution of Eq.(3.4) fails to be compatible.

[3] We are grateful to C. Vafa for pointing this out.

The above equations freeze $h_{1,1}$ special coordinates at constant vacuum expectation values[4] depending on the intersection numbers, fluxes and the constant ζ. The latter is not independent: ζ can be expressed in terms of the intersection numbers and fluxes by using the constraint (3.13). For the purpose of illustration, we discuss below two simple examples.

Example 1: $h_{1,1} = 1$, $\mathcal{V}(S) = S^3$. This is a model without vector multiplets, for example a quintic CY. There is a trivial solution

$$X^S = 1 \quad , \qquad \zeta = \alpha_S \ . \tag{3.15}$$

Example 2: $h_{1,1} = 2$, $\mathcal{V}(S,T) = ST^2 - \frac{1}{3}T^3$. $X_{12}(1,1,2,2,6)$ CY with one vector multiplet and a flop transition [16,17]. A simple calculation yields

$$X^S = \frac{\zeta(\alpha_S + \alpha_T)}{3\alpha_S(\alpha_T - \alpha_S)} \ , \qquad X^T = \frac{2\zeta}{3(\alpha_T - \alpha_S)} \ , \qquad \zeta = \frac{3}{2^{2/3}}\alpha_S^{1/3}(\alpha_T - \alpha_S)^{2/3} \ . \tag{3.16}$$

For generic fluxes $\alpha_T > \alpha_S > 0$, this is a regular solution valid in the Kähler cone $S > T$. However, if $\alpha_T = \alpha_S$, it is pushed to the flop at $S = T$.

The above solution has previously been obtained in [8] (although written in a different parameterization) by solving the supersymmetric Killing equations with constant vector moduli, $(X^i)' = 0$.[5] Our derivation utilizes the field equations and yields the same result, although the starting point, Eq.(3.3), is a weaker Ansatz than $(X^i)' = 0$. We will be using these field equations in Section 5 to obtain some information on the tension of three-brane sources, without assuming that these sources preserve bulk supersymmetry.

It is also worth mentioning that Eq.(3.14) has a nice interpretation in terms of very special geometry: the surface $\mathcal{V}(X) = 1$ tends to align in such a way that its normal vector becomes parallel to the flux vector α_i [9].

To summarize, we obtain a Poincaré three-brane solution which, for generic values of background fluxes, freezes the vector moduli fields at constant vacuum expectation values, fixing the shape of Calabi-Yau manifold. On the other hand, the hypermultiplet modulus that determines the volume becomes a linear function of the transverse coordinate u, see Eq.(3.10). There is an inevitable singularity at $u = -\mathcal{V}_0/\zeta$, where the Calabi-Yau manifold shrinks to a point. Then the three-brane Weyl factor also vanishes, see Eq.(3.5), therefore the whole $D = 10$ spacetime collapses to one point. We will be revisiting this problem later.

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[4] Of course, this is provided that a solution exists.

[5] Ref.[8] contains also a class of solutions involving u-dependent moduli.

4. The Attractor Connection

It is well known [14,15,18] that the entropy of five-dimensional BPS black hole solutions of $N = 2$ Einstein-Maxwell supergravity is determined by the extremal value of the central charge. This value is attained at the horizon which from the point of view of the vector moduli space acts as an attractor point. Such black holes appear in CY compactifications of M-Theory and their entropy can be computed at the microscopic level by counting the number of M2-branes wrapping around CY two-cycles [19]. We will now show that in the Poincaré three-brane solution the vector moduli are frozen at exactly the same attractor point, with the fluxes α_i identified as BPS charges.

First, note that homogeneity of the volume, Eq.(2.2), together with Eq.(3.9) imply that

$$\zeta = \alpha_i X^i(M) = Z[X(M)] \, , \tag{4.1}$$

where Z is the central charge for a BPS state with electric charges α_i [16]. Given this, Eq.(3.9) can be written as

$$\frac{\partial Z}{\partial M^i}[X(M)] = 0 \, . \tag{4.2}$$

This means that the vector moduli X^i are frozen at the extremum of the central charge. It is also clear that the constant ζ is equal to the extremal value of the central charge. The above equations can be rewritten in terms of very special geometry, without referring to the underlying moduli M, as the familiar [15] $D = 5$ attractor stability condition:

$$D_i Z = 0 \quad , \quad Z(X) = \alpha_i X^i \quad , \quad \zeta = Z\big|_{D_i Z = 0} \, , \tag{4.3}$$

where we used the covariant derivative

$$D_i = \frac{\partial}{\partial X^i} - \frac{1}{6} c_{ijk} X^j X^k \tag{4.4}$$

appropriate for $Z(X)$ defined on the surface $\mathcal{V}(X) = 1$.

We conclude that in the presence of a Poincaré three-brane, the vector moduli are forced to the same attractor configuration as on the horizon of a charged black hole. This indicates that charged black holes may play an important role in resolving the zero volume singularity.

5. Three-brane Sources

We can avoid the singularity if we consider other flux configurations. We will discuss fluxes jumping across one or two three-brane sources. Their tension will be determined below. Similar ideas have been discussed before in various contexts in several places, including [20,3,6,9]. We consider the cases of non-compact and compact transverse dimension separately.

5.1. Non-compact transverse dimension: a single brane source

Let us consider a three-brane located at $u = 0$, with the flux jumping from $-\alpha_i$ for $u < 0$ to $+\alpha_i$ for $u > 0$. The scalar potential does not change upon reversing the flux direction, therefore the bulk action remains the same as in Eq.(2.8). Similarly, the solution of Section 3 remains valid for $u > 0$. In order to obtain a solution for $u < 0$ it is sufficient to change the signs $\alpha_i \to -\alpha_i$ and $\zeta \to -\zeta$. Hence the moduli remain frozen at the same attractor point as before, see (3.14), and the Weyl factor is still given by Eq.(3.5). On the other hand, the CY volume[6]

$$\mathcal{V} = \mathcal{V}_0 + \zeta |u| \ . \tag{5.1}$$

It is clear that the zero volume singularity can indeed be avoided if $\zeta > 0$ (for $\zeta < 0$ one would have to reverse the flux directions).

The cusp at $u = 0$ contributes additional terms proportional to $\delta(u)$ to the field equations. Therefore, in order to obtain a self-consistent solution valid everywhere in $D = 5$ spacetime, the bulk action must be supplemented by a term of the form

$$S_{\text{brane}} = - \int d^4 x \sqrt{-g^{(4)}} f(M) \ , \tag{5.2}$$

representing an explicit three-brane source with efective tension $f(M)$ at $u = 0$. The four-dimensional metric $g^{(4)}_{\alpha\beta}$ is induced by the bulk metric $g_{\mu\nu}$: $g^{(4)}_{\alpha\beta} \equiv \delta^\mu_\alpha \delta^\nu_\beta g_{\mu\nu}(u = 0)$. The moduli-dependent three-brane tension $f(M)$ is constrained by the field equations in the following way. The $(\alpha\beta)$ components of Einstein's equations require

$$f = -\frac{\zeta}{\mathcal{V}_0} \ . \tag{5.3}$$

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[6] The additive integration constants have been adjusted to ensure that the volume is a continuous function of u.

On the other hand, the moduli field equations dictate

$$\frac{\partial f}{\partial M^i} = 3\mathcal{V}_0^{-4/3}\alpha_i \ . \tag{5.4}$$

It is easy to find a function that, with the help of Eqs.(3.8) and (4.1), satisfies these constraints:

$$f(M) = -\mathcal{V}^{-4/3}\alpha_i M^i \ . \tag{5.5}$$

If one returns to the string frame by undoing the Weyl rescaling (2.7) so that $\sqrt{-g^{(4)}} \to \mathcal{V}^{4/3}\sqrt{-g^{(4)}}$, then the three-brane tension becomes

$$f_s(M) = -\alpha_i M^i \ = -\int_{[G]} J \ , \tag{5.6}$$

where $[G]$ is the two-cycle Poincaré dual to the four-form field strength G_{flux} of Eq.(2.5):

$$\int_{[G]} \omega_i = \int_{CY} G_{\text{flux}} \wedge \omega_i = \alpha_i \ . \tag{5.7}$$

Note that the required tension is negative. Its magnitude however is equal to the tension of a fivebrane wrapping a CY two-cycle. The string theory origin of such objects is likely to be found in the framework of F-theory compactifications [21].

5.2. Compact transverse dimension: a pair of branes with opposite tension

Let us assume that the transverse dimension is compactified on a circle, with $u \in (-1, 1]$. Starting from the non-compact domain wall solution discussed before, we can construct a simple periodic configuration with the flux changing direction $(\alpha_i \to -\alpha_i)$ at $u = 0$ and then reversing back to its original value at $u = 1$. The CY volume $\mathcal{V}(u)$, Eq.(5.1), now becomes a periodic function zigzagging between $\mathcal{V}_0$ and $\mathcal{V}_0 + \zeta$. The additional cusp at $u = 1$ forces us to introduce another flux source. By repeating the previous arguments one can identify this source as a brane with the tension $\widetilde{f} = -f$. This brane could be identified with an M-theory fivebrane wrapping a CY two-cycle. In this way, the pair of branes with opposite tension supports a flux configuration in the compactified space.

This solution is somewhat similar to the configuration studied by Randall and Sundrum [10]. There is however no room in M-Theory for a fine-tuning of cosmological constants: the bulk vacuum energy originates from G-fluxes while tensions of the effective three-brane sources are determined by the Calabi-Yau geometry. As a result, one obtains a Weyl factor which is different from AdS-like exponential warp factors that localize

gravity. Furthermore, one would expect that the equilibrium of brane configurations considered here is not stable under small perturbations[7]. All these points deserve further investigation.

6. Supersymmetry

In this Section, we examine the supersymmetry transformations in order to determine what (if any) type of supersymmetry is preserved by our solutions. To that end, it is convenient to use the notation of [22], with the two $N = 2$ supersymmetry generators labelled by $\pm$. In the gravitational background (1.1), the non-vanishing components of the spin connection can be rewritten using Eq.(3.5) as

$$\omega^{au} = (\mathcal{V}^{1/6})' dx^a , \tag{6.1}$$

where a denotes the $D = 4$ Lorentz indices. Thus the supersymmetry variations of the gravitinos become

$$\delta\psi_\alpha^\pm = 2\partial_\alpha\eta^\pm \pm i\frac{\zeta}{6\mathcal{V}}\sigma_\alpha\left(\frac{\mathcal{V}'}{\zeta}\bar\eta_\mp + \bar\eta_\pm\right) ,$$
$$\delta\psi_u^\pm = 2\partial_u\eta^\pm + \frac{\zeta}{6\mathcal{V}}\eta^\mp , \tag{6.2}$$

where we used Eq.(4.1). In order to find the unbroken supersymmetries we first set these variations to zero and solve the corresponding Killing spinor equations.

For the solution of Section 3, i.e. in the absence of brane sources, $\mathcal{V}' = \zeta$, and the Killing equations are solved by

$$\eta_0^+ = -\eta_0^- = \epsilon\mathcal{V}^{1/12} , \tag{6.3}$$

where ϵ is a constant Weyl spinor.

If a source is inserted at $u = 0$, as in the examples discussed in Section 4, then $\mathcal{V}' = \text{sgn}(u)\zeta$, and

$$\eta^+ = \epsilon\mathcal{V}^{1/12} \quad , \quad \eta^- = -\epsilon\mathcal{V}^{1/12}\text{sgn}(u) . \tag{6.4}$$

However, in this case

$$\delta\psi_u^- = 4\epsilon\mathcal{V}_0^{1/12}\delta(u) , \tag{6.5}$$

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[7] In principle, this problem could be circumvented by working on an orbifold S^1/Z_2 and placing the sources at the fixed points.

hence the Killing equations are satisfied everywhere in the bulk but they are *not* satisfied on the brane hypersurface. Furthermore, it is easy to see that for the respective solutions, the spinors (6.3) and (6.4) give vanishing supersymmetry variations of all other fermions: hyperinos and gauginos. In particular, the gaugino variations vanish for the moduli frozen at the attractor point (4.3).

In this way, we reach the conclusion that the singular solution preserves $N = 1$ supersymmetry. The regular solutions involving brane sources preserve $N = 1$ supersymmetry in the bulk, however they break it on the branes as long as $\mathcal{V}_0 \neq 0$.

7. Conclusions and Outlook

In this work we studied $D = 5$ Calabi-Yau compactifications of M-Theory with background G-fluxes and explicit effective three-brane sources. In the absence of sources there exists an $N = 1$ supersymmetric solution with the metric representing a Poincaré three-brane. The Weyl factor depends on the transverse coordinate u as $(\mathcal{V}_0 + \zeta u)^{1/3}$. At $u = -\mathcal{V}_0/\zeta$ the three-brane as well as the CY manifold shrink to a point. The vector moduli that determine the shape of the Calabi-Yau manifold remain frozen at a point similar to the well-known black hole attractor point, which suggests that charged black holes play an important role in resolving the zero volume singularity.

The singularity can be avoided altogether by introducing a G-flux source along the three-brane hypersurface. In this case, the CY volume reaches its minimum at the position of the source. Although the string theory origin of such a source remains unclear, its tension is equal (up to a sign) to the volume of the two-cycle dual to the flux vector α_i. If the transverse dimension is compactified on a circle, one can also construct a brane configuration similar to the Randall-Sundrum configuration. In M-Theory though, the bulk vacuum energy is completely determined by G-fluxes and the brane tensions by CY geometry – as a result one obtains a gravitational background which is not of the AdS-type.

There is a number of points deserving further investigation. We raised several stability issues. In the solutions involving three-brane sources, the minimum value of CY volume remains undetermined at the level of classical field equations. The obvious question is how this zero mode, and the solution in general, are affected by quantum corrections. Furthermore, one should analyze the stability of brane configurations with respect to their relative positions. It is possible that answers to these questions can be obtained in the framework of F-theory (or its orientifold limits).

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